

NONLINEAR EVOLUTION OF R-MODES IN ROTATING RELATIVISTIC STARS

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Abstract. A numerical study of nonlinear r -modes in isentropic, rapidly rotating relativistic stars, via 3-D general-relativistic hydrodynamical evolutions, is presented. On dynamical timescales, we find no evidence for strong coupling of r -modes to other modes at amplitudes of order one or larger. Therefore, unless nonlinear saturation sets in on longer timescales, the maximum r -mode amplitude is of order one. An absolute upper limit on the amplitude is set by causality. Our simulations also show that r -modes and inertial modes in isentropic stars are discrete modes, with no evidence for the existence of a continuous part in the spectrum.

1. Introduction

The study of the properties of r -modes in rotating compact stars and their relevance to relativistic astrophysics has received considerable attention since the discovery (Andersson 1998, Friedman & Morsink 1998) that these modes are unstable to the emission of gravitational radiation. The r -mode instability provides an explanation for the spin-down of rapidly rotating young neutron stars to Crab-like spin-periods and for the spin-distribution of millisecond pulsars and accreting neutron stars. Additionally, it is considered to be a strong source of continuous gravitational radiation (see Friedman & Lockitch 1999 for a review). Moreover, if r -modes induce differential rotation, their interaction with the magnetic field in neutron stars can en-

hance the toroidal magnetic field of the star (Rezzolla, Lamb & Shapiro 2000).

Before the instability can have an effect on the spin evolution of a young neutron star, the r -mode grows to an amplitude where it is saturated by nonlinear effects. Motivated by the absence of studies of such nonlinear saturation we performed a numerical study of r -mode hydrodynamical evolutions in rapidly rotating relativistic stars. We tried to elucidate what is the maximum amplitude such modes can reach, before nonlinear saturation (via hydrodynamical coupling) sets in (Stergioulas & Font 2000). The present contribution highlights the main findings of our study. We note that the saturation is most likely to set in on a hydrodynamical timescale, although it cannot be excluded that weak hydrodynamical couplings saturate the r -mode amplitude on longer timescales (but shorter than the growth timescale due to gravitational radiation reaction). However, at present, those long timescales cannot be achieved accurately in 3-D simulations, even with the largest available supercomputers.

2. Numerical framework

For our study we use a numerical code based on the 3-D **CACTUS** code (Font et al 2000, Alcubierre et al 2000), into which we implemented the 3rd order PPM method (Colella & Woodward 1984) for the hydrodynamics, and initial data for equilibrium and perturbed rapidly rotating relativistic stars (Stergioulas & Friedman 1995). In Font, Stergioulas & Kokkotas (2000) it was shown that the PPM method is suitable for long-term evolutions of rotating relativistic stars. We focus on a representative, rapidly (and uniformly) rotating model with gravitational mass $M = 1.63M_{\odot}$, equatorial circumferential radius $R = 17.25\text{km}$ and spin period $P = 1.26\text{ms}$. We use the $N = 1.0$ relativistic polytropic equation of state. Our simulations employ 116^3 Cartesian grid-points, yielding a resolution of 0.31km per zone.

The excitation of r -modes is achieved by perturbing the initial stationary model, adding a specific perturbation δv^i to the contravariant components of the equilibrium 3-velocity, v^i . As there is no exact linear eigenfunction available in the literature that would correspond to an $l = m = 2$ r -mode eigenfunction for rapidly rotating relativistic stars, we use an approximate eigenfunction, valid in the slow-rotation $O(\Omega)$ limit to the first post-Newtonian order (Lockitch 1999).

We also note that since r -modes are basically fluid modes we only evolve the hydrodynamical variables, keeping all spacetime variables fixed at their initial, unperturbed, values. The computational requirements for an accurate, coupled spacetime and hydrodynamical evolution, by far exceed current supercomputing resources.

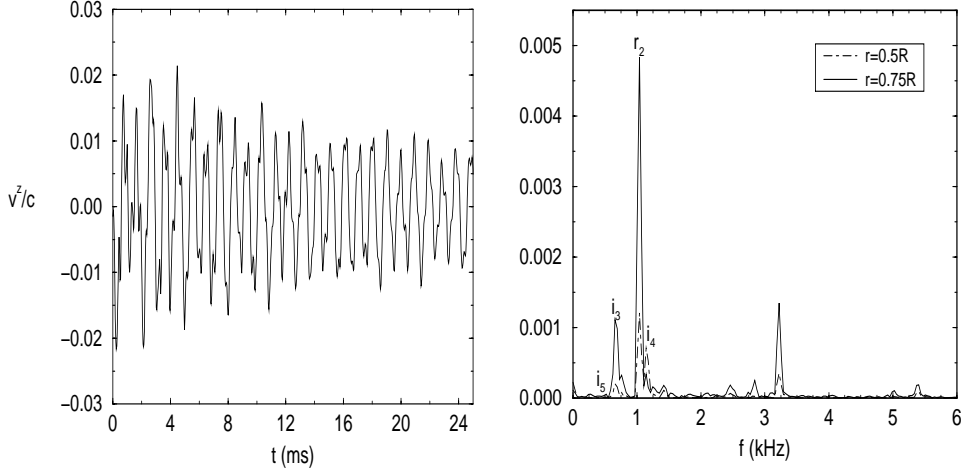


Figure 1. *Left:* Evolution of the axial velocity in the equatorial plane for an amplitude of $\alpha = 1.0$, at $r = 0.75R$. The evolution is a superposition of (mainly) the $l = m = 2$ r -mode and several inertial modes. The amplitude of the oscillation decreases due to numerical (finite-differencing) viscosity of the code. *Right:* Fourier transform of the velocity time-evolution, showing the frequencies of the modes in the inertial frame. The frequencies are the same at different radii, which implies a discrete spectrum.

3. Discussion

Figure 1 (left panel) displays the evolution of the axial velocity in the equatorial plane (v^z along the y -axis) at a radius of $r \sim 0.75R$. The evolution is a superposition of several modes, the $l = m = 2$ r -mode being the dominant component. The chosen amplitude of the eigenfunction is $\alpha = 1.0$ (in the Newtonian limit, our definition of the amplitude agrees with that of Owen et al 1998). The perturbed star is evolved for more than 25ms (26 $l = m = 2$ r -mode periods), during which the amplitude of the oscillation decreases due to numerical viscosity only. With much larger amplitudes the evolution is still similar to that in Figure 1, with no sign of nonlinear saturation of the r -mode amplitude on a dynamical timescale. Therefore, unless nonlinear saturation sets in at timescales much longer than the dynamical one, unstable r -modes could be driven to large amplitudes (of order one) by gravitational radiation reaction. An absolute upper limit on the r -mode amplitude is set by causality, requiring $\sqrt{v_i v^i} < c$.

A Fourier transform of the time-evolution, as a function of the frequency in the inertial frame, is shown in the right panel of Figure 1. It reveals that the initial data we are using, excite mainly the $l = m = 2$ r -mode (r_2), with a frequency of 1.03 kHz and, with smaller amplitudes, several inertial

modes (i_3 , i_4 , i_5) and higher harmonics. The frequencies of the various modes are the same at any given point inside the star, which indicates that the spectrum is a sum of discrete modes, in agreement with the conclusions of Lockitch, Andersson & Friedman (2000).

We have also investigated the possible appearance in our evolutions of the kinematical drift reported by Rezzolla et al (2000). In a rotating star with a poloidal magnetic field, this kinematical drift may wind up the magnetic field lines and limit the effect of the r -mode instability. Our results do indicate that the perturbed star is rotating slower near the surface (compared to the unperturbed star) and that this drift scales roughly as α^2 for amplitudes of order $\alpha \sim 1.0$, although its magnitude is significantly smaller than that estimated by Rezzolla et al (2000).

Our finding that gravitational radiation could drive unstable r -modes to a large amplitude implies that r -modes can easily melt the crust in newly-born neutron stars (Lindblom, Owen & Ushomirsky 1999), leaving the initial conclusions about the r -mode instability being a strong source of gravitational waves (see Friedman & Lockitch 1999) essentially unchanged.

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